Overview of Methods for Magnetic Susceptibility Measurement

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Abstract— In this paper, an overview of methods for magnetic susceptibility measurement is described. Older methods — Faraday's scale and Guoy's scale are based on force effects of magnetic filed to magnetized specimen. Another methods — Inductive methods, use change of coil inductance, when magnetically conductive specimen is embedded. Modern methods (SQUID magnetometer) benefits from quantum interference device, allowing such sensitive magnetic measurement, that magnetic quantum can be detected. Magnetic resonance is another modern way, how to measure susceptibility and some of MR based methods were introduced. The authors of this article mainly focus on the measurement of magnetic susceptibility of non-ferromagnetic material by means of MRI methods. In this respect, three basic measurement techniques are known and covered in this article.

1. INTRODUCTION

Magnetic susceptibility is the physical quantity describing material properties in the external magnetic field [1]. Magnetic susceptibility is defined as ratio between magnetization **M** of the material in the magnetic field and the field intensity **H**:

$$\mathbf{M} = \chi_{\mathrm{m}} \mathbf{H}. \tag{1}$$

All materials can be classified by value of magnetic susceptibility into three groups:

- diamagnetic materials: $-1 < \chi_{\mathrm{m}} < 0$,
- paramagnetic materials: $0 < \chi_{\mathrm{m}} \ll 1$,
- ferromagnetic materials: $\chi_{\mathrm{m}} \gg 1$.

Several methods are used for magnetic susceptibility measuring such as Faraday's scale, Guoy's scale or inductive method with SQUID magnetometer. For detailed description of these methods see [2]. The MR based method for susceptibility measurement makes also this measurement possible [3, 4].

2. FARADAY'S SCALE

The Faraday's scale (Fig. 1(a)) is suitable for susceptibility measurement of a small specimen made from paramagnetic, diamagnetic or even ferromagnetic materials. When inserting the specimen of volume V with total magnetic moment

$$\mathbf{M}_{\mathrm{C}} = \mathbf{M} \cdot V \tag{2}$$

into magnetic field, energy change occurs

$$E = -\frac{1}{2}\mu_0 \mathbf{M}_{\mathrm{C}} \mathbf{H}. \tag{3}$$

Force acting on the sample in magnetic field with gradient in direction x is [5]

$$F = -\frac{dE}{dx} = \frac{1}{2}\mu_0 V \frac{\mathrm{d}\left(\mathbf{M}_{\mathrm{C}}\mathbf{H}\right)}{\mathrm{d}x} = \mu_0 \chi_{\mathrm{m}} V H \frac{\mathrm{d}H}{\mathrm{d}x}, \tag{4}$$

for linear dependence of this force on susceptibility value we need gradient field meeting the condition:

$$H \frac{dH}{dx} = konst. \tag{5}$$

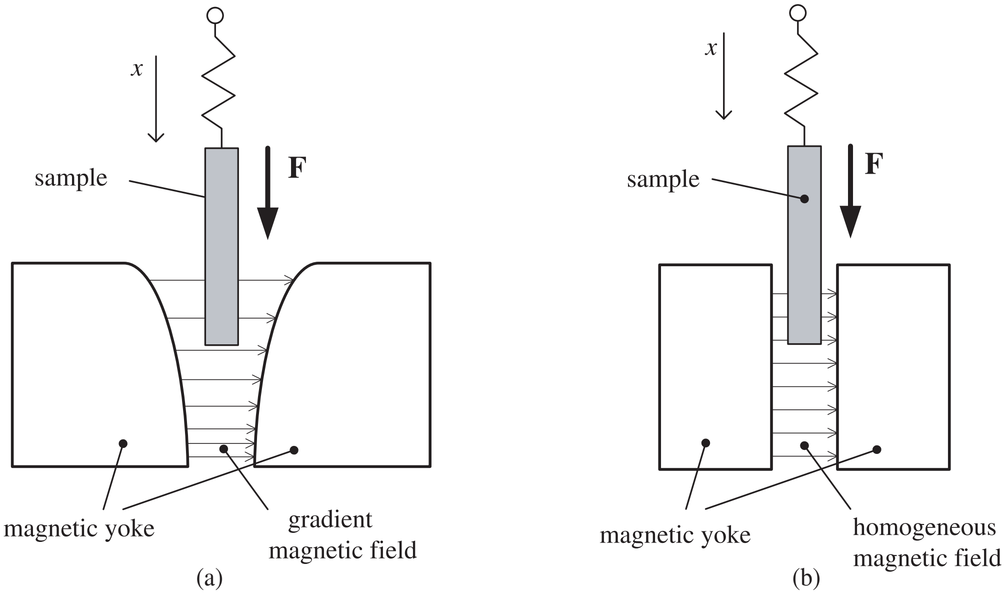

Figure 1: Principle draft of (a) Faraday's scale and (b) Gouy's scale used for susceptibility measurement.

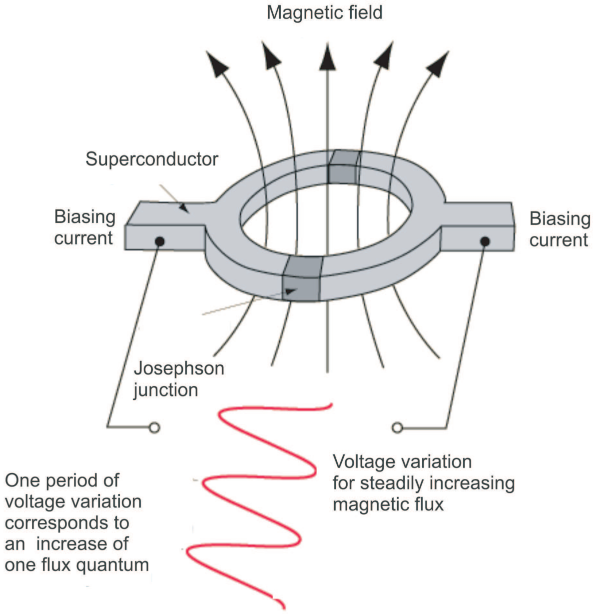

Figure 2: Principle of SQUID magnetometer.

3. GUOY'S SCALE

This scale uses slightly modified principle compared to Faraday's scale. Homogenous magnetic field is used instead of the gradient one (Fig. 1(b)). Axial force acting on the sample in magnetic field is

$$F = \frac{1}{2}\mu_0 \chi_{\mathrm{m}} S \left(H_1^2 - H_2^2\right), \tag{6}$$

where S is cross-section of sample in x direction and H_1, H_2 are magnetic field intensities in inner and outer end of the sample. The achieved sensitivity of magnetic susceptibility measurement is 10^{-9} with accuracy of 1%.

4. INDUCTIVE METHOD

Induction method is based on change of coil inductance invoked by embedded specimen. Unbalanced bridge of two identical coils powered by stable harmonic current generator is used, where one coil has reference yoke and specimen is inserted into the second one. This kind of susceptibility evaluation methods is obviously used in geology measurement.

5. SQUID MAGNETOMETER

The superconducting quantum interference device (SQUID) can be used as an extremely sensitive detector of magnetic flux. It consists of two parallel Josephson junctions — Fig. 2. The great sensitivity of the SQUID devices is associated with measuring changes in magnetic field related to one flux quantum

$$\Phi_0 = \frac{h}{2e} = 2,068 \cdot 10^{-15}\,(\mathrm{T} \cdot \mathrm{m}^2). \tag{7}$$

If a constant biasing current is maintained in DC SQUID, the measured voltage oscillates with change in the magnetic flux. Counting the oscillations allows evaluating the flux change which has occurred. Because of the necessary superconductive state, this device works only at low temperatures (4.2 K, liquid helium).

6. MR METHOD

The authors of this article mainly focus on the measurement of magnetic susceptibility of non-ferromagnetic material by means of MRI methods [1]. In this respect, three basic measurement techniques are known. The first was described by Wang [6], who characterized an MRI susceptibility measurement method which utilizes a resonant frequency discontinuity at the interface between two materials, each having an observable MR signal. The susceptibility difference between the two materials can be obtained using the data acquired from the vicinity of the interface without knowing all details of the geometry of a sample.

The second method of magnetic susceptibility measurement in samples either assumes a uniform susceptibility distribution or further requires a well-defined geometric shape [7, 8]. A voxel-based inversion requiring a sufficient number of measurement points was proposed [9, 10]. However, the inversion is computationally intensive and no experimental work applying this technique has been published to date. The numerical difficulty may be sidestepped by recasting the inverse problem as an iterative model fitting problem, but such a solution underestimates the susceptibility by 50%. The magnetic field map interpolation as a means for image correction is also utilized by Sumanaweera [11]. The inverse problem is further complicated by the nonuniform noise in the field measurement and by the high phase noise in regions with strong susceptibility due to signal voids caused by T_2^* effects. Another disadvantage of this method consists in the necessity to have a sufficiently large number of measured points. Here, it is important to note that these techniques are based on the knowledge of the map of magnetic field inside a sample (thus, the sample must be magnetically compatible).

The third (and a very interesting) approach to magnetic susceptibility measurement was described in [1, 2]. In these papers, the authors inquire into the calculation of magnetic susceptibility. A sample of a weakly magnetic material embedded in a magnetic field causes a distortion of the static magnetic field. The susceptibility of a sample material can be computed from the shape of this reaction field in the vicinity of the sample. In contrast to the method described above, in this way it is possible to measure materials which do not provide any MR signal. In [1], an analytical calculation of the reaction field is derived using a numerical model and the method of boundary elements. The susceptibility of a sample is calculated from this reaction field. The calculation of magnetic susceptibility is limited to the infinitely large plane of a sample.

6.1. Magnetic Susceptibility Measurement from 3D Map of Reaction Field

This method of susceptibility measurement is based on the assumption of constant magnetic flux in the working space of superconducting magnet. Inserting a specimen with magnetic susceptibility χ_s causes local deformation of previously homogeneous magnetic field — for illustration see Fig. 3.

The magnitude of these deformations depends on the difference of magnetic susceptibility of the specimen χ_s and of its vicinity χ_v, on the volume and shape of the specimen, and on the magnitude of basic field B_0.

Let there be a static magnetic field described by B_0, both in the z direction. Assume that a cylindrical specimen of diameter d and length l ($l_\mathrm{s} \gg d$) is inserted into the magnetic field, parallel to the direction of B_0. The behaviour of magnetic induction $B_\mathrm{z}(x)$ in the position $y = 0$ and $z = 0$ on a straight line is shown in Fig. 3. The difference between the change in the magnetic field in the specimen vicinity and the value of static magnetic field B_0 is called the reaction field ΔB.

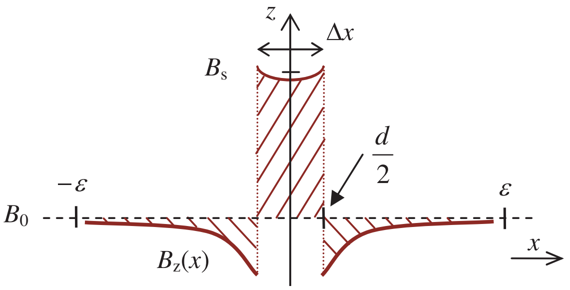

Figure 3: Idealized shape of magnetic flux density $Bz(x)$ in paramagnetic specimen and its vicinity.

As can be seen, the specimen affects the field not only in its volume but also in its vicinity. Magnetic flux density inside the specimen will, according to [1], be equal to:

$$B_{\mathrm{s}} = B_0 \left(1 + \chi_{\mathrm{s}}\right). \tag{8}$$

Assume a constant magnetic flux Φ through the normal area of cross-section S_{z} of the magnet working space [8]:

$$\Phi = \iint_{S_{\mathrm{z}}} B \cdot \mathrm{d}S = \mathrm{const.} \tag{9}$$

from which it is evident that magnetic flux density outside the specimen is changed, resulting in a shape that can be considered the superposition of homogeneous field B_0 and reaction field ΔB.

From Eqs. (8) and (9) we derived the following relations for the calculation of magnetic susceptibility from a 3D map of the reaction field being measured [1]:

$$\chi = -\frac{\iiint\limits_{\Omega} \Delta B_{\mathrm{v}} \mathrm{d}x \mathrm{d}y \mathrm{d}z}{V_{\mathrm{s}} \cdot B_0}, \tag{10}$$

where ΔB_{v} is reaction field in the vicinity of specimen, V_{s} is volume of the specimen and B_0 is a static magnetic field.

7. CONCLUSIONS

The methods of magnetic susceptibility measurement are clearly described in this article. The authors of this articles focus on the magnetic susceptibility measurement from the 3D reaction field. This reaction field was measured by the NMR tomograph. The methods of NMR magnetic susceptibility measurement are more detail in references [1].

ACKNOWLEDGMENT

The work described in the paper was financially supported by the projects GA102/1/0318, CZ.1.05/2.1.00/01.0017 (ED0017/01/01) and FEKT-S-11-5/1012.

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